Nash Equilibrium

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Abstract:

This entry defines the concept of a Nash equilibrium that is central to understanding the predictions of game theory. It considers the applications of Nash equilibrium for the study of the theory of oligopolistic competition.

Definition:

A Nash equilibrium of a game is a set of strategies undertaken by agents such that no agent can improve their payoff by choosing another strategy holding the strategic choices of all other agents fixed.

Cross references:

Game theory
Oligopoly
Commitment

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Nash Equilibrium

Game theory is a branch of applied mathematics that economists and, now, researchers and practitioners in strategic management have adopted to understand observed choices of firms and other agents in market environments. Central to the predictive power of game theory is the identification of equilibrium outcomes – or solution concepts – in game representations of strategic environments. The primary concept of equilibrium employed for this purpose is Nash equilibrium. An "equilibrium" is usually defined as a point of rest. A point of rest in a game is naturally defined as an outcome where there is no tendency for change. As changes in games are brought about by changes in strategic choice of agents playing a game, it is natural to match the equilibrium of a game with the tendency of agents to believe that they cannot improve their own payoff by changing their strategies. Nash equilibrium provides the most direct means of making that evaluation.

Here we briefly provide a historical overview of the concept of Nash equilibrium before turning to its formal definition. We then examine refinements that have allowed researchers to focus attention of particular Nash equilibria with distinct and potentially more natural properties. Finally, a review of the use of Nash equilibrium in modelling oligopolistic behaviour is undertaken.

Brief historical overview

While the concept of Nash equilibrium was first formally stated and analyzed by John Nash (who shared a Nobel Prize in economics for this achievement in 1994) – see Nash (1950, 1951) – the concept was originally applied by Antoine Augustin Cournot in 1838. There he considered two firms who were faced with the choice of the quantity of output to supply in a market. These choices interacted with an increase in one firm's output changing market price and with it the margins the other firm would earn by producing more. To sort through this interaction, Cournot looked for an equilibrium point where each firm was choosing the output that maximized its own profits on the assumption that the output of its rival was fixed. Then, taking the conditions for maximization and ensuring that they were reconciled – that is, that one firm's assumption of the other firm's output matched the choice that firm would actually make (in mathematics, formally, looking for a fixed point) – Cournot found the equilibrium levels.

Nash's contribution was to modernise the approach of Cournot for the new mathematical theory of games as exposited by John von Neumann and Oskar Morgenstern (1944). They had noted that strategies of agents in a game might be pure or mixed. The latter involved a randomisation over a set of actions that might be chosen by a player. They showed that for zero-sum games (where one agent only wins at the expense of another) that an equilibrium existed in either pure or mixed strategies. Nash was able to demonstrate this existence for general games (both zero and non-zero sum).

Formal definition of a Nash equilibrium

To formally define a Nash equilibrium, one must start with a game. A game is comprised of a set of agents, N with element n, and for each agent, n, they have a set of strategies X_n from which they can select one, x_n; although that can select a randomisation over those strategies as part of a strategy profile. For a given set of selected strategies, $\{x_n\}_{n \in N}$, each agent, n, receives a payoff $\pi_n(x_n, x_{-n})$ where x_{-n} is the set of selected strategies of agents other than n.

Games can have different forms. They can be ones where each agent chooses their strategy after observing some of the strategies of other agents (i.e., sequential move games). They can be ones where agents never observe some strategies of other agents (i.e., games of incomplete information). Or they can be ones where agents never observe any of the strategies of other agents before committing to their own. These final games are called simultaneous move games.

In a simultaneous move game, a pure strategy Nash equilibrium is a set of strategies, x_n^* for each agent, *n*, such that:

$$\pi_n(x_n^*;x_{-n}^*) \geq \pi_n(x_n;x_{-n}^*) \text{ for all } n \text{ and for all } x_n \neq x_n^*$$

A Nash equilibrium that allows for mixed strategies is defined analogously but with expected payoffs over potentially randomised strategy profiles. The formal definition captures the notion that a set of strategies is an equilibrium if, holding the strategic choices of other agents as fixed, no agent wants to choose an alternative strategy.

Types of Nash equilibrium

There is rarely a unique Nash equilibrium in a game. This is especially so in sequential move games and games with incomplete information but can also arise in simultaneous move games. In some situations that multiplicity of equilibria represents an interest prediction of games. For instance, Thomas Schelling (1978) demonstrated this with respect to coordination failures that has since been applied in strategic management to understand network effects and platform strategy (see, for example, Shapiro and Varian, 1998).

In other situations, the multiplicity of Nash equilibria arises because the concept of Nash equilibria has too little structure to identify more plausible equilibria. In sequential move games, this arises when a Nash equilibrium outcome of a game involves a strategy that comprises a threat that is not credible. For instance, an incumbent in a market may want to play a strategy that involves setting a very low price (perhaps below marginal cost) should an entrant incur sunk costs to enter the market. There is often a Nash equilibrium in such games that involves the entrant not entering as a result of a forecast of that low price. However, the incumbent's pricing threat may be non-credible in the sense that, should an entrant actually enter, the incumbent might no longer find it worthwhile, from that point on, to price low. If the entrant sees through this, the entrant will enter and the unique equilibrium outcome will involve the entrant entering and competitive pricing emerge. Reinhart Selton (1975) studied this problem and developed a refinement to Nash equilibrium termed *subgame perfect equilibrium* to require that all threats (and indeed promises) in sequential move games be credible.

Modelling Oligopoly

In strategic management, Nash equilibrium is applied wherever game theory is used. This is particularly the case when it comes to modelling the behaviour of oligopolistic firms. As noted above, a natural way of modelling this comes from assuming that firms can commit to quantities leading to a Cournot Nash equilibrium. However, it can equally be the case that competing firms can be modelled as competing in prices. This leads to a Bertrand equilibrium outcome (usually involving price at short-run marginal cost or with a mark-up when there is some product differentiation). But the choice between them is not always a free one and can relate to the underlying characteristics of the industry (Ghemawat, 1997). In addition, the type of strategic variables that are the focus of competition can also impact on the strategic incentives to engage in other activities (e.g., advertising, R&D and entry); see Fudenberg and Tirole (1984). This has meant that significant care must be taken when applying game theory to generate predictions for empirical testing (Sutton, 1991).

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